siderable density, in which the testing-tube has to rotate; and in calculating the degree of force which exists in the blood, as shown by the rapidity of the movements of the tube under the influence of the magnet, allowance must be made for the resistance from this source that has to be overcome: consequently any fibrin that has not been removed must, in the same degree in which it increases the viscidity and density of the fluid, increase the resistance to the movements of the tube, and so interfere with the manifestation of results.

2nd. By preserving the blood from contact with the atmosphere any change in its physical character from such source is prevented.

3rd. And for the reason that the paramagnetic force in arterial blood must depend upon the amount of oxygen it contains, and the diamagnetic force in venous blood upon carbonic acid, it is evident that the force in the testing-tube as opposed to the force in the suspending medium must be very little, whilst the mechanical resistance afforded by such medium must be considerable. It is therefore essential that the battery-force should be of sufficient power to *develop* these forces to the greatest possible extent.

Addendum.

Since writing the foregoing paper, in repeating the experiments, it has been found that, for the due performance of them, the blood should be maintained as nearly as possible at its natural temperature. To effect this the shape of the vessel (No. 2) has been altered, and it is immersed in a water-bath, the heat of which is sustained by a spirit-lamp, its temperature, and also that of the blood, being regulated by thermometers placed in each vessel.

VI. "On the Multiplication of Definite Integrals." By W. H. L. Russell, F.R.S. Received October 28, 1874.

The definite integral $\int_{y_0}^{y_1}\int_{x_0}^{x_1} \mathrm{P}\,dx\,dy$ may be considered geometrically as the integral $\int \mathrm{P}\,dx\,dy$ extended over an area bounded by the straight lines whose equations are

$$x=y_1,\quad x=y_0,\quad y=x_1,\quad y=x_0.$$

Now conceive the axes transformed through an angle of 45°, so that $x=\frac{\xi}{\sqrt{2}}-\frac{\eta}{\sqrt{2}}$, $y=\frac{\xi}{\sqrt{2}}+\frac{\eta}{\sqrt{2}}$; then the equations to the four straight lines become

$$\frac{\xi}{\sqrt{2}}-\frac{\eta}{\sqrt{2}}=y_1,\quad \frac{\xi}{\sqrt{2}}-\frac{\eta}{\sqrt{2}}=y_0,\quad \frac{\xi}{\sqrt{2}}+\frac{\eta}{\sqrt{2}}=x_1,\quad \frac{\xi}{\sqrt{2}}+\frac{\eta}{\sqrt{2}}=x_0;$$

and computing the integral as extended over an area bounded by the four straight lines thus represented, we have

$$\int_{y_0}^{y_1}\int_{x_0}^{x_1} \mathrm{P}\,dx\,dy = \int_{\frac{x_0+y_0}{\sqrt{2}}}^{\frac{x_0+y_1}{\sqrt{2}}}\int_{y_0\sqrt{2}-\xi}^{\xi-x_0\sqrt{2}} \mathrm{P}\,d\eta\,d\xi + \int_{\frac{x_0+y_1}{\sqrt{2}}}^{\frac{x_1+y_0}{\sqrt{2}}}\int_{y_0\sqrt{2}-\xi}^{y_1\sqrt{2}-\xi} \mathrm{P}\,d\eta\,d\xi$$

$$+\int_{\frac{x_1+y_0}{\sqrt{2}}}^{\frac{x_1+y_1}{\sqrt{2}}}\int_{\xi-x_1\sqrt{2}}^{y_1\sqrt{2}-\xi} \mathrm{P}\,d\eta\,d\xi.$$

After I had discovered this formula, I found that it had been already given in a memoir by Dr. Winckler in the Vienna Transactions for 1862. This memoir treats of the transformation of double integrals between fixed limits, and seems to me one of great interest and importance. My present object is to give two formulæ for the multiplication of definite integrals which will not be found in Dr. Winckler's paper.

$$\int_{y_0}^{y_1} \epsilon^{y^2}dy\int_{x_0}^{x_1} \epsilon^{-x^2}dz = \frac{\epsilon^{-x_0^2}}{2}\int_{y_0}^{y_1}\frac{\epsilon^{z^2}dz}{z+x_0} - \frac{\epsilon^{-x_1^2}}{2}\int_{y_0}^{y_1}\frac{\epsilon^{z^2}dz}{z+x_1}$$

$$-\frac{\epsilon^{y_0^2}}{2}\int_{x_0}^{x_1}\frac{\epsilon^{-z^2}}{z+y_0} + \frac{\epsilon^{y_1^2}}{2}\int_{x_0}^{x_1}\frac{\epsilon^{-z^2}dx}{z+y_1}.$$

Also

$$\int_{y_0}^{y_1} \epsilon^{y^2}dy\int_{x_0}^{x_1} \epsilon^{x^2}dx = \frac{x_1\epsilon^{x_1^2}}{2}\int_{y_0}^{y_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+x_1^2} - \frac{x_0\epsilon^{x_0^2}}{2}\int_{y_0}^{y_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+x_0^2}$$

$$+\frac{y_1\epsilon^{y_1^2}}{2}\int_{x_0}^{x_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+y_1^2} - \frac{y_0\epsilon^{y_0^2}}{2}\int_{x_0}^{x_1}\frac{dz\,.\,\epsilon^{z^2}}{z^2+y_0^2}.$$

The use of these formulæ is easily seen.

December 17, 1874.

JOSEPH DALTON HOOKER, C.B., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers and Communications were read:—

I. "On Polishing the Specula of Reflecting Telescopes." By W. Lassell, F.R.S., V.P.R.A.S. Received November 11, 1874.

(Abstract.)

The object of this paper is to describe a method of giving a high lustre and true parabolic curve with ease and certainty, by appropriate machinery, to the surfaces of the specula of large reflecting telescopes.